

Dr. G. Rebel
6 Calcot Court
Calcot
Reading
RG31 7RW
United Kingdom

Tel : +44-118-942-3238
Mobile : +44-797-343-1367
Email : gerhard@bgisl.com
Web : <http://www.bgisl.com>



OTC 12173

Tension/Torsion Interactions in Multicomponent Mooring Lines

C.R. Chaplin, G. Rebel and I.M.L. Ridge, University of Reading

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This paper was prepared for presentation at the 2000 Offshore Technology Conference held in Houston, Texas, 1-4 May 2000.

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Abstract

Six strand ropes generate torque under tension. This can lead to transfer of twist to other mooring line components, either permanently, or dynamically. The effects of this imposed twist can seriously affect strength and fatigue endurance. In order to predict the torsional interactions between components it is necessary to quantify the tension/torsion behavior of all the different categories of mooring line component, not just the six strand rope. This paper discusses the problems of torsional interaction and presents results of measurements of tension/torsion behavior of six strand rope, stud link chain and a PET fiber rope.

Introduction

Mooring systems for floating facilities are becoming more complex, especially for floating production systems placed in deeper water and with the adoption of PET components. One feature of these trends is a tendency to combine different types of component in a single "composite" line, whether this be a temporary expedient to facilitate deployment, or a final assembly. It is useful to identify four different categories of component which are used:

- chain;
- six (or possibly eight) strand wire rope;
- spiral strand; and,
- PET fiber rope (of various constructions).

These different classes of component share properties which make them suitable for use in mooring systems: they all have high axial strength and stiffness combined with low bending stiffness. However, for any given axial strength the different categories have very different weights (especially when submerged), they have different costs, different degrees of "ruggedness" and exhibit some different axial stiffness properties. These differences are what lead to the selection of

components for different elements in a composite mooring line. But there is another category of mechanical response that comes as a consequence of the geometry and material properties of these components: torsional behavior.

The torsional stiffness, tension/torsion response, and sensitivity to twist differ appreciably between these different components. This is especially important due to the particular behavior of six strand rope which is used both as an installed component as well as for work wires and pendants during deployment: six stranded rope is not "torque balanced" which means that under tensile loading, if restrained, it will develop an axial torque, but if unrestrained it will untwist to maintain zero torque. This torque reaction is caused by the helical nature of the rope's construction. Because of this somewhat unusual behavior it is important to understand the torsional restraint presented to six strand rope by the other components to which it might be attached in a mooring line. This is necessary to predict the transfer of twist from one component to another, and its consequences.

These problems have been a pre-occupation of the rope research group at the University of Reading for the past few years, and some aspects of the group's studies have been published previously^{1,2,3}. This paper discusses some of the torsional mechanisms which seem to be increasingly prevalent offshore; presents details of the measured response of different components at laboratory scale; and, sets out methodologies for making quantitative predictions of these effects.

Characteristic Torque Properties of Components

Six strand wire rope has a pronounced torque reaction when tensioned. Its torsional stiffness is non-linear, increasing with tension, and further increasing when twisted up (lay length decreasing) while decreasing when untwisting (lay length increased). Though moderately tolerant of permanent torsional distortion, cyclic twisting in phase with axial fatigue can be very damaging.

Spiral strand is basically torque balanced compared to six strand rope. It has a higher torsional stiffness than six strand rope, and like six strand its torsional stiffness increases with axial tensile loading. However it is vulnerable to imposed twist which can adversely affect the load balance between different layers of wires and so impair fatigue performance.

Stud link chain has a highly non-linear behavior. It is well balanced torsionally. At near zero tensile loading it has a very

low torsional stiffness but stiffness increases very rapidly with tension to be far higher than either six strand or spiral strand ropes. The torsional stiffness is also a function of rotation being low initially, then increasing rapidly until knots form and further torque increase is effectively limited (for any given tension), but provided failure does not take place first. This low initial stiffness is often viewed as capacity to accommodate limited rotation without any effect, which is put at around 3° per link. Any such limit is a function of detailed link geometry as well as being influenced by line tension. No information has been found in the public domain on the torsional properties of chain.

PET fiber rope is a more complex issue as (like wire rope) it can be manufactured in different constructions to have different levels of torque balance. However the small diameter of the basic fiber means that, whatever the construction, it will have a low torsional stiffness, and should be tolerant of imposed twist.

The elastic stability of long thin members under combined torque and axial load is complex^{4,5}, but high torque under limited tension can lead to the formation of a hockle (or loop) which mathematicians call an *elastica loop*. The hockle formation is driven by exchange of elastic energy between different forms of deformation (torsion, extension and bending). Hockles therefore generally only form when tension is very low: so a moderate tension will prevent such occurrences. If torsional stiffness is low there is little driving force and the tendency to hockle formation is further reduced: this is the case with fiber ropes. Once a hockle has formed in a wire rope the effect of retensioning is generally severe local distortion, with significant consequences for the load sharing between wires that seriously damages fatigue performance (to a greater extent in spiral strand than six strand).

Eccentric deformations can also occur in chain in the form of "knotting" where the chain twists up upon itself, one link at a time. This is a local instability induced by applied rotation, which normally occurs only in slack chain.

Torsion Mechanisms in Offshore Moorings

There are two quite different categories of torsional interaction. The first is interaction between components during deployment, which can result in net rotations being introduced into the permanently installed line. Secondly where installed components have different characteristics the point of connection can be forced to rotate in phase with any tensile load fluctuation to maintain torque equilibrium.

Twist introduced during installation

When a six strand rope is under tension it will untwist if the rotation is not restrained. Raising and lowering heavy chains or anchors can induce moderately high tensions in pendant ropes or work wires used for the purpose. Two examples illustrate the issues:

1. An anchor, attached to a mooring chain, or rope, is lowered by a six strand pendant, and the chain is lying slack on the seabed. The chain offers negligible restraint to rotation and the pendant progressively untwists as it is

paid out. The twist accumulates in the slack chain. After the anchor has been positioned, and the pendant buoyed off, the buoy rotates to restore the torque in the unloaded pendant to equilibrium, but the twist in the chain remains to be redistributed as the line is tensioned – being passed to components having a lower torsional stiffness if present.

2. An anchor has been pre-installed with a length of ground chain which is attached to a pendant connected to a buoy. To couple the next element in the line, the anchor handling vessel picks up the buoy, which is disconnected and the pendant winched in to raise the chain. As the heavy chain is taken up by the rope, tension increases and with the chain slack on the seabed, the rope untwists and twist accumulates in the chain. The end of the chain is recovered, the pendant is disconnected and the chain attached to the next element of the permanent mooring system, say a spiral strand. The spiral strand is paid out and the assembly lowered to the seabed. When the line is tensioned the twist is thrown forward from the chain into the spiral strand which, under load, has a much lower torsional stiffness. When the spiral strand is slackened off it forms hockles or corkscrew distortions which cause serious loss of fatigue endurance.

Torsional interactions between installed components

Components connected in series which are all torque balanced, and where there is no net rotation resulting from installation should not exhibit rotation under load, even if the components have a different torsional stiffness.

A similar set of torque balanced components with different torsional stiffness, and with rotation which has been induced during deployment will probably show some dynamic rotation as the system attempts to maintain torque equilibrium throughout. This kind of dynamic rotation might be exacerbated by inertial effects, where heavy couplings or the components themselves acts as a "flywheel" and torsional oscillations are excited*.

When one of the components in the line is not torque balanced and other components are not stiff enough to restrain the torque generated, rotations will occur.

Three examples illustrate the problems further:

1. A six strand rope is coupled to a chain. Under the action of line tension the wire rope generates torque but the tension is always sufficient to ensure that there is no rotation at the connector: no problems.
2. A six strand rope is connected to a chain via a "perfect swivel" (i.e. one which can rotate under high tension and with negligible torque reaction). The line tension will fluctuate in response to vessel motions in the normal way, but the swivel provides no torque reaction, allowing the

* It is worth noting here that the torsional stiffness of most of the components with which we are concerned here is very low so that, in comparison to extensional modes, the natural frequencies of torsional oscillation is low and may under some circumstances converge with wave periods.

rope to rotate in response to every change in line tension so as to retain zero torque. This is fine for the chain, but causes a real problem for the wire rope.

3. A deep water mooring system employs a conventional drag embedment anchor connected to a ground chain through the touch-down zone, to a torque balanced PET fiber rope, then finally via a six strand rope to conventional deck mounted winches. The PET rope has such a low torsional stiffness especially in view of the length in deep water that it offers little torsional restraint. Indeed in practice it might approximate to that "perfect swivel" so the connector between wire rope and fiber rope rotates to achieve zero torque.

Predicting Behavior

It is clear from the discussion above that to understand these interactions and to have any chance to make quantitative predictions it is necessary to have quantitative tension/torsion data for ALL of the different types of component that might be employed.

Torsional issues in wire ropes have been the subject of numerous previous investigations. Kollros⁶ has developed a simple two parameter model for torsional behavior, while Feyrer & Schiffner⁷ developed a set of more complex models which takes account of the increase in stiffness with tension, for a range of rope constructions. Rebel⁸ has made extensive tests of triangular strand ropes from which to model the rotation of hoisting rope in deep shafts; this approach includes modeling of the initially high torsional stiffness as friction is overcome. Raoof & Hobbs⁹ have derived and validated models for the torsional behavior of spiral strand, and Kraincanic & Hobbs¹⁰ have made similar comparisons for six strand rope.

The ABS guidance notes¹¹ on synthetic ropes outline methods for testing the torque balance of fiber ropes, but do not provide for a comprehensive evaluation of torsional stiffness. Otherwise there are models for predicting fixed end torque reaction in fiber ropes¹².

As already indicated above, no published information has been found on the subject of the torsional behavior of chain.

Tension/Torsion Testing

Accurate modeling of the torsional interaction between mooring components requires that the torsional properties of the components are quantified. In general the preferred (most accurate) method for determining these properties is by means of physical tension/torsion testing.

Fig. 1 shows typical tension/torsion test results for a 19 mm RHO IWRC wire rope, a 34 mm parallel laid, twisted-strand polyester fiber rope with a braided external jacket and a 20.5 mm stud link chain. All three components have a breaking load (BL) of about 250 kN. The twist (°/m) is determined by dividing the applied end rotation by the original gauge length in the untwisted state. These curves for constant twist are generated by inducing rotation into (or removing rotation from) the sample and then holding this rotation while a tensile load is applied and the tension and torque monitored.

From measured test data, it is possible to develop empirical equations which define the relationship between tensile load and torque at various values of constant twist. Such relationships then form the basis for predicting the behavior of single or combined components under operating conditions. It can be seen from the results in Fig. 1 that torsional properties can be quite different which explains some of the unexpected behavior which has been observed when such components are connected together and tensioned offshore.

Tension/Torsion Fatigue Issues

Recent investigations at Reading have demonstrated that combined tension/torsion loading of six strand wire ropes can severely detriment the rope performance¹. For a given cyclic load range, the fatigue life of the rope is significantly influenced by the amplitude of torsional twisting (°/m). Fig. 2 shows tensile fatigue endurance, referred to the rope life with no cyclic twist. The load range for these tests was from 8% to 39% (actual breaking load) using 19 mm RHO IWRC wire rope connected to either a 34 mm strength-matched polyester fiber rope or a tapered roller thrust bearing.

Further investigations are required to determine the nature of the transition between zero cyclic twist and 200°/m. Of particular importance to mooring systems is the effect which rotating masses can have on wire rope performance. The only difference in test conditions between the two right hand data points in Fig. 2 was the inertia of the rotating coupling attached to the bearing. Increased inertia resulted in increased cyclic twist which reduced life by a factor of 2. If, for instance, a short section of heavy chain was connected between a six strand wire rope and polyester fiber rope in a composite mooring line, this would increase the detrimental effects of any tension/torsion coupling between the wire and fiber ropes.

Installation Case Study

In this case the amount of rotation transferred into a slack ground chain is considered, as a function of water depth, and with two different pendant rope diameters.

A simple rope torque model is considered based on the work of Feyrer & Schiffner⁷ in which axial torque, *M*, is given by:

$$M = c_1 Fd + c_2 Fd^2 \frac{d\phi}{dz} + c_3 Gd^4 \frac{d\phi}{dz} \dots\dots\dots (1)$$

where *c*₁, *c*₂ and *c*₃ are constants, which have been determined for different rope constructions; *d* is rope diameter; *G* is wire shear modulus; *F* is tension; *φ* is rotation; and *z* rope length. For the ropes the following properties have been taken:

construction	6 x 36 with IWRC	
diameter	64 and 52 mm	
mass in seawater	10 and 6.6 kg/m	
<i>c</i> ₁	0.085	from Ref. 7
<i>c</i> ₂	0.187	from Ref. 7
<i>c</i> ₃	0.000531	from Ref. 7
<i>G</i>	75,000 N/mm ²	

Note in the expression for torque and twist, units of N and mm have been used throughout and twist ($d\phi/dz$) is defined in radians/mm.

For the chain, properties assumed are:

diameter	102 mm
mass in seawater	199 kg/m
pitch of links	408 mm

Calculations have been made for a range of water depths to a maximum of 2000 m. It is assumed that at the outset the pendant extends from a surface buoy to the chain on the seabed which must be lifted to the surface by winching in the pendant. It is assumed that the chain on the seabed is quite slack, and the AHV moves astern maintaining the chain in a slack state so that no torsional restraint is offered. The weight of the suspended mass gives rope tension, and by equating torque to zero in Eq. 1, twist can be calculated. By repeating for each increment wound onto the drum, the (opposite) twist transferred to the chain can be derived.

The untwisting of the pendant rope transferred to the AHV winch drum corresponds to the rotation imparted to the chain. In the calculation it is assumed that the chain length is some 50 m greater than the depth. In this hypothetical scenario it is further assumed that connection of the chain to whatever other components might be in the mooring line does not allow for removal of the twist from the chain. However chain can typically absorb 3° per link. Fig. 3 shows the residual, or excess, twist in the chain after deducting this 3° per link.

It is clear that the residual twist in the chain increases rapidly with depth, and that the smaller diameter rope untwists more under the same loading. Under load a chain in this twisted state when coupled to a rope as part of a mooring system will throw the twist into the more flexible rope. The twisted rope is then vulnerable to hockling if unloaded.

The values chosen in this case study do extend to the extreme: 2000 m of suspended chain is a lot. At these extreme depths the resultant number of turns is such that knotting of the chain would occur, which would restrict further rotation, as well as introducing another damage mechanism to threaten line integrity.

Interaction case study

Using the measured tension/torsion characteristics of mooring components it is possible to calculate the rotation which would occur between two components with dissimilar tension/torsion properties when connected in series and tensioned, as in Fig. 4.

For the analysis in this section, it is assumed that the rotation boundary conditions BC1 and BC2 in Fig. 4 are both zero. The properties of both components have been modeled as described in the Appendix to predict the equilibrium torque under an axial tension of 100 kN.

Fig. 5 shows the torque measured as a function of load applied to a 19 mm RHO IWRC wire rope in series with a 34 mm polyester fiber rope for two different length ratios (fiber to wire rope). The ends of both ropes were prevented from rotating but the connection between them was allowed to rotate freely to achieve torque balance. The length ratios for

each case are shown in the figure, as are the calculated equilibrium torques at 100 kN.

The calculation is in three stages:

1. The connection between the rope is assumed to be restrained from rotation and the torque in the wire rope calculated for the 100 kN applied tensile load, using Eq. A-1.
2. The connector is then 'released' and the change in torque calculated which will bring the torques in both components to the same magnitude (but opposite sense) to achieve equilibrium, using Eq. A-2a and A-1.
3. Using the change in torque the twist of the wire rope is calculated using Eq. A-6.

The initial torque (step 1) for the lower curve in Fig. 5 is 112 Nm, and the calculated change in torque is - 82 Nm. This brings the assembly to the equilibrium torque of 30 Nm (step 2). The calculated rotation of the connector is 272° (step 3) which compares favorably to an observed rotation of 250°. It is interesting to note that the value calculated using Eq. A-6 is much closer to the observed value than the rotation implied in the 'release' process of stage 2, which was 409°. The reason for this is that Eq. A-6 captures the increased torsional stiffness of the rope due to inter-wire friction which is not reflected in the data in Fig. 1(a) or in Eq. A-1.

These calculations have demonstrated a method for determining the rotation which would occur between tension components with different torsional properties which are connected in series. The main input data to the calculations are the tension/torsion properties of the components (Fig. 1) and where wire ropes and chain are concerned, the torque-twist stiffness properties which include friction effects (Eq. A-6).

Discussion

The connection in series of components which have different tension/torsion characteristics can result in rotation at the connection. Where this happens whilst handling of the components during deployment, damaging levels of permanent twist can result, possibly transferred from one component to another. In many cases it is possible to make quantitative predictions of the amount of torque that can be introduced, but such predictions are dependant upon mathematical models of the complex characteristics of the components involved. Some information is available for stranded wire ropes and most of that relates to much smaller diameters than are typical of offshore operations. There is even less data available for the tension/torsion properties of the nominally torque balanced components (fiber ropes, spiral strand ropes and chain). This paper has demonstrated the nature of this problem and the form of the characteristics of a components in each of the major categories. Calculation techniques appropriate to these issues have been described, with different degrees of sophistication to match the level required by any particular problem.

The concern where deployment procedures induce twist (typically via slack chain), is that subsequently hockling can develop. It is the straightening of the hockled rope that

degrades the integrity of the mooring line.

These problems become more acute with greater depth, and greater mismatch in properties.

Where the torsional interaction occurs between installed mooring line components having mismatched characteristics a different danger may exist. Experiments at laboratory scale have demonstrated that when the torque generated in a six-stranded rope is not restrained, and rotation allowed, there can be a serious loss of fatigue endurance¹. This loss of endurance appears to be a function of the degree of torsional restraint. Quasi-static methods for predicting this effect have also been demonstrated, but seem sensitive to the accuracy of the tension/torsion modeling.

Dynamic rotation in phase with line tension fluctuations can be influenced by other considerations such as inertia. The quantitative prediction of such dynamic phenomena requires determination of hysteresis as well as non-linear stiffness.

The precise relationships between rotational restraint, and tensile fatigue loading levels has yet to be established, even at small scale. The loss of fatigue endurance observed in the laboratory scale tests reported here is sufficiently great to be a matter of considerable concern. The possibility of such an effect has been known for some time; but that it could be so great has not previously been reported.

One postscript of relevance concerns the use of swivels when coupling mismatched components. This has yet to be investigated experimentally, but the majority of such devices will not rotate when transmitting significant levels of tension, but do serve to make handling easier by allowing release of local twist when disconnecting assemblies, say on the deck of an AHV. But should a genuinely low friction swivel be deployed, whilst it might help to avoid some of the installation types of problem described above, it might make the coupled tension/torsion fatigue worse, providing even less restraint than, say, a fiber rope.

Conclusions

1. Interactions between mooring line components with mismatched tension/torsion characteristics can result in significant amounts of axial twisting, either permanent or dynamic.
2. Static twist can result in hockles that when pulled out impair fatigue performance.
3. Dynamic twisting of a six-strand wire rope can very seriously reduce fatigue endurance.
4. Predictions of torsional interaction can be made but are dependent upon accurate measurement to characterize behavior, both in terms of torque developed as a function of tension at different levels of rotation, but also torsional stiffness at different tensions and starting twist.
5. There is a need for component characterization at full scale, for which the authors are planning manufacture of a suitable facility.
6. Finally, an awareness of the nature of these interactions, is the most important factor in designing mooring systems and their deployment procedures to minimize the effects.

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Acknowledgements

The authors wish to acknowledge the financial support of sponsors of the Reading Rope Research group: Amerada Hess Ltd., BPA, Bridon International and Petrobrás. Rope test specimens were kindly supplied by Brugg Wire Ropes Inc. and Marlow Ropes Limited.

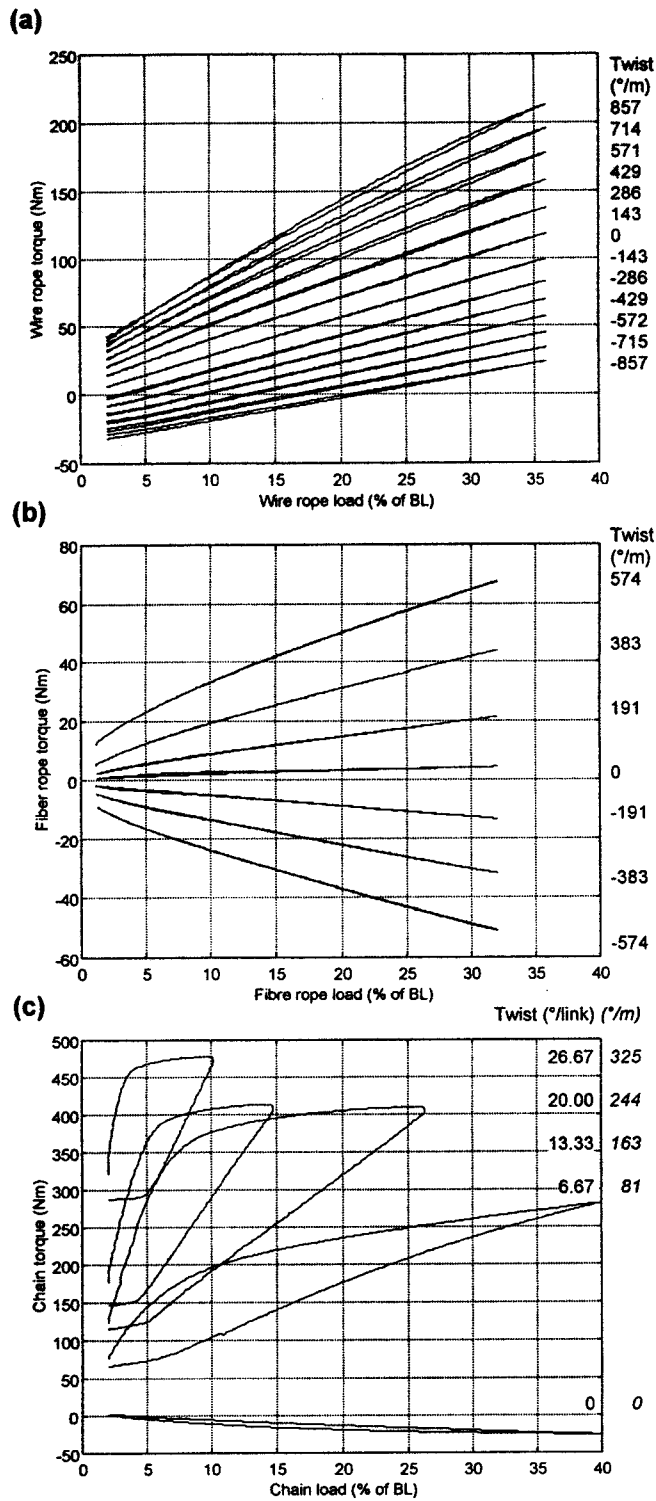


Fig. 1—Torsion/tension test data for (a) 19 mm RHO IWRC rope; (b) 34 mm parallel twisted-strand polyester fiber rope; and (c) 20.5 mm stud link chain. All three components have a breaking load (BL) of about 250 kN. Note that the y-axis scales on the three graphs have different ranges.

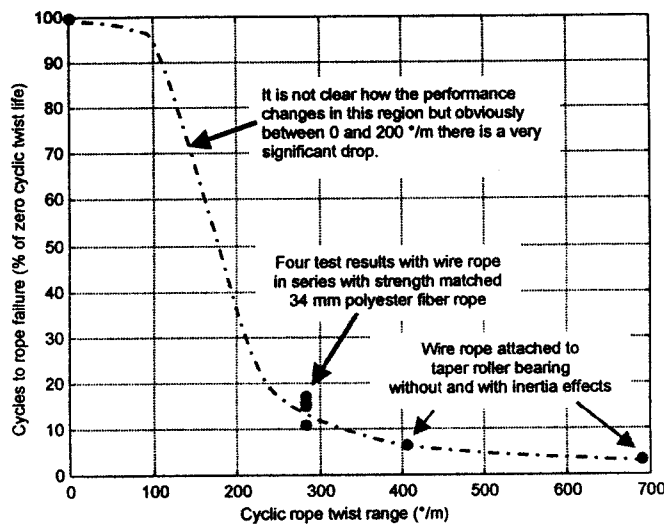


Fig. 2—Tension/torsion fatigue test data for a 19 mm 6x19 RHO IWRC wire rope, connected in series with 34 mm parallel twisted-strand polyester fiber rope or thrust bearing.

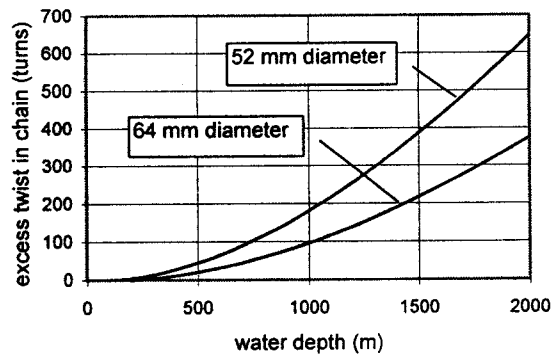


Fig. 3—Excess twist transferred to 102 mm chain by 52 mm pendant (upper line) and 64 mm pendant (lower line).

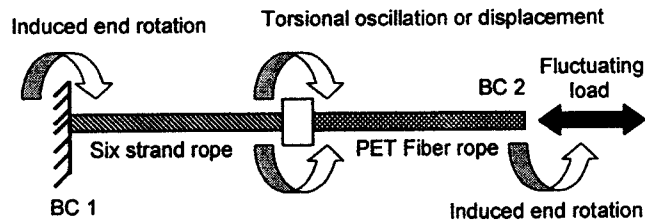


Fig. 4—Schematic representation of mooring system components with different torsional properties connected in series.

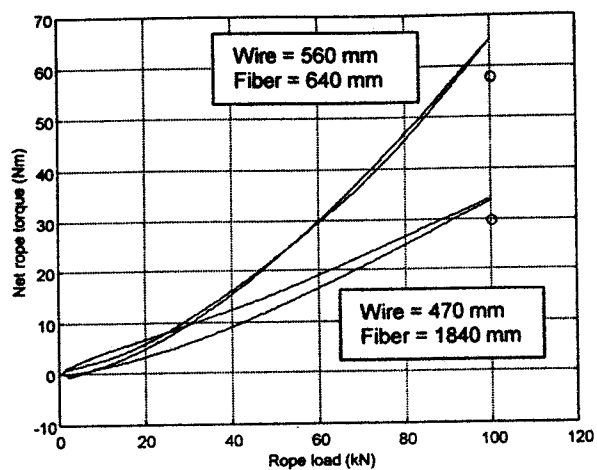


Fig. 5—Measured rope torque for a 19 mm RHO IWRC wire rope connected in series with a 34 mm polyester fiber rope. The circles indicate the calculated equilibrium torques from Eq. A-2a (in the Appendix).

Appendix—Calculation of Torsional Interaction

To calculate the rotation at the connection between components as shown in Fig. 4 it is necessary to define an equation for the torque in the wire and fibre ropes. Rebel⁸ showed that for six strand wire ropes the following equation can be used to approximate the tension/torsion characteristics as shown in Fig. 1(a). Since the fiber rope test data (Fig. 1(b)) has a similar general form (curve shape) the equation is also suitable for describing its response. Thus:

$$M = \begin{bmatrix} R^2 & R & 1 \end{bmatrix} \cdot \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{bmatrix} \cdot \begin{bmatrix} F^2 \\ F \\ 1 \end{bmatrix} \dots\dots\dots(A-1)$$

where :

M = rope torque (Nm) F = rope tensile load (kN)
 $R = \phi / l =$ rope twist ($^{\circ}/m$) ϕ = rope end rotation ($^{\circ}$) l = rope length at zero twist (m)
 N = constants determined from 2nd order least squares fit to test data

Assuming zero rotation boundary conditions at each end of the assembly, for equilibrium at a given constant tension (F) the torque (M) in the wire rope must equal the torque in the fiber rope. Since the wire rope generates a far higher torque at zero twist than the fiber rope it will unwind and will therefore be expected to see a negative end rotation ($-\phi$) :

$$M(\text{wire rope}) = M(\text{fiber rope}) \dots\dots\dots(A-2)$$

$$\begin{bmatrix} \left(\frac{-\phi}{l_{\text{wire}}}\right)^2 & \frac{-\phi}{l_{\text{wire}}} & 1 \end{bmatrix} \cdot \begin{bmatrix} N_{\text{wire}} \\ F \\ 1 \end{bmatrix} = \begin{bmatrix} \left(\frac{\phi}{l_{\text{fiber}}}\right)^2 & \frac{\phi}{l_{\text{fiber}}} & 1 \end{bmatrix} \cdot \begin{bmatrix} N_{\text{fiber}} \\ F \\ 1 \end{bmatrix} \dots\dots\dots(A-2a)$$

After solving Eq. A-2a for the rotation seen by both ropes at the coupling, ϕ , the equilibrium torque is found by substituting ϕ back into the torque equation for either the wire rope or fiber rope (Eq. A-1). However since torque versus tension and connection rotation measurements were taken during the tension/torsion fatigue tests described earlier, it was possible to compare the calculated equilibrium torque and associated connection rotation at a given load with the measured / observed values (Fig. 5).

The following constants were used in the calculations:

$$F = 100 \text{ kN} \dots\dots\dots(A-3)$$

$$N_{\text{wire}} = \begin{bmatrix} -2.043\text{e-}9 & 5.226\text{e-}7 & -4.034\text{e-}6 \\ -3.172\text{e-}6 & 1.121\text{e-}3 & 3.830\text{e-}2 \\ -7.528\text{e-}4 & 1.195\text{e+}0 & 6.501\text{e-}5 \end{bmatrix} \dots\dots\dots(A-4)$$

$$N_{\text{fiber}} = \begin{bmatrix} -1.898\text{e-}9 & 2.806\text{e-}7 & 2.399\text{e-}6 \\ -4.194\text{e-}6 & 1.422\text{e-}3 & 1.322\text{e-}2 \\ -2.970\text{e-}4 & 7.079\text{e-}2 & 4.088\text{e-}3 \end{bmatrix} \dots\dots\dots(A-5)$$

In a separate torsion test, at constant load, it was determined that the wire rope torque decreases with untwisting of the rope as described by Eq. A-6:

$$\Delta M = a \cdot \arctan(b \Delta R) + c \Delta R \dots\dots\dots(A-6)$$

with constants:

$$a = 0.55 \quad b = 0.23 \quad c = 0.14$$

Once the change in torque (ΔM) is determined from the initial zero twist value of torque in the wire rope at 100 kN (Eq. A-1) and the equilibrium torque after rotation (Eqs. A-2a and A-1) the actual twist to achieve the change in torque (ΔR) can be calculated using Eq. A-6.